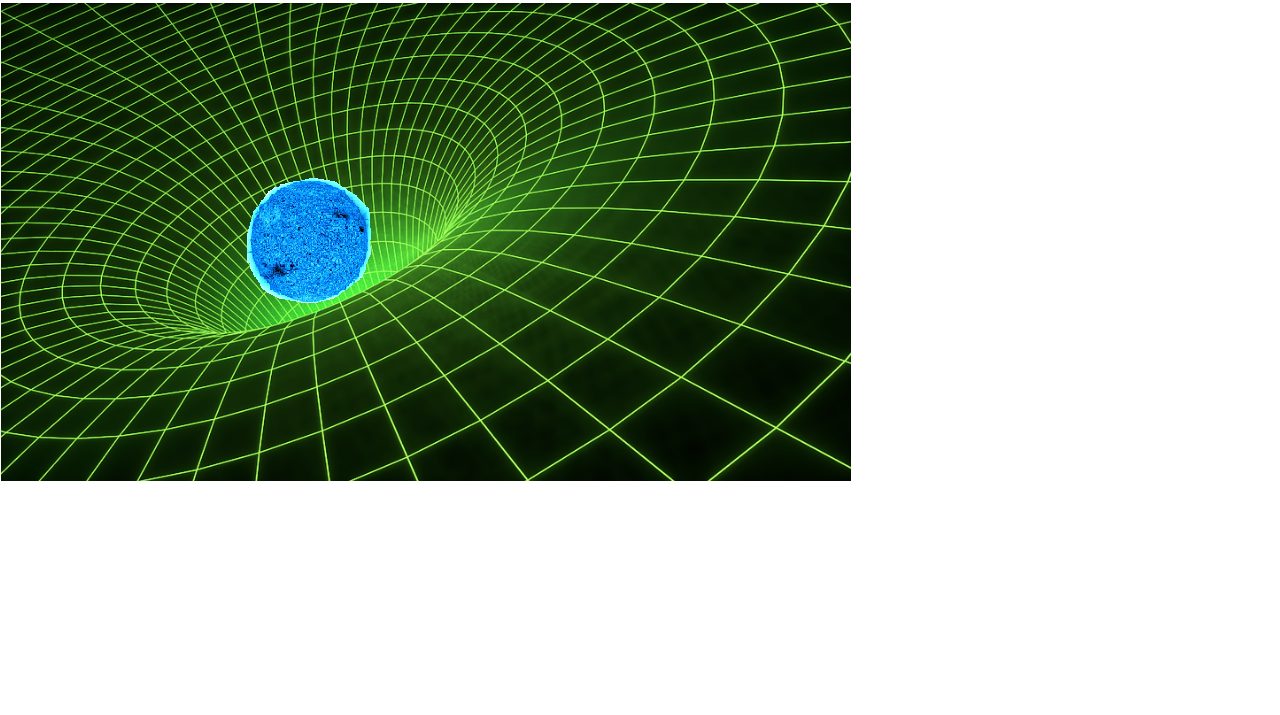
**Stars Exterior**

Now let’s explore another special case – stars. More generally, we’ll simply look at spherically symmetric arrangements, but these always describe stars.



Again we’ll start with Einstein’s equations:



Since stars are spherically symmetric, we will assume spherical symmetry in our metric ansatz. Additionally, we’ll assume that in our reference frame, the metric is independent of time, and also possesses time-reversal symmetry so that the star basically isn’t rotating. Let 0 go in the time-like direction, r go in the radial direction, θ in the theta directions, and φ in the phi direction. In that case we can write our metric as:



Note that t, r, θ, φ are coordinates in the time-like and radial-like, and theta-like, phi-like directions. But the former two are not identical to physical time and physical radial distance of course. What is the LHS of the Einstein equation? Let’s work it out. Our metric tensor is:



where T = T(r), R = R(r), Θ = r2, Φ = r2sin2θ, and e00, e11, e22, e33 are just place holders indicating the position of the elements in the matrix. Proceeding,



and



and so we should have for Γ,



The Ricci tensor is:



Now taking the derivative and trace at the same time, to form the first term in the Ricci tensor we have:



Next,



Next,



Next, and finally,



And now adding these together…we have:



Grouping together,



In particular, we’ve got:



The next term is:



and the next,



and the R33 guy,



The off-diagonal elements are zero, as can see…



and,



So so far, our metric is:



and our curvature tensor is so far:



and let’s get the curvature scalar:



So we have:



Now we must fill these expressions into the Einstein equation.



**Exterior solution**

Well, we’ll use the other form of Einstein’s equations – see Geometry file:



Now, if we’re looking for solutions outside the star, then Tμν = 0. And so our equations reduce to simply, setting Λ = 0 as well,



Let’s try to solve these equations (the last equation is dependent on the others and doesn’t yield a new condition).



Now I guess of these three equations, only two are strictly linearly independent, since we have only two uknown functions, R and T. The first thing we might try to do is eliminate the Tr2 terms from the top two equations. So let’s multiply the top by R/T and subtract it from the bottom. This will give us:



These equations imply that TR = constant. Let’s denote this constant by α. Then we have T = α/R. Now substituting this equation into the equation for R22 we get:



where k is some constant to be determined. It follows therefore that:



Now we must work out what k and α are supposed to be. Let’s compare to the weak field limit of Einstein’s equations:



and of course, using the φ = -GMm/r for a star, far from a star we should get that



This suggests that we have α = –c2, and k = -2GM/c2. If so then we have the following metric outside a star,



where,



We see that when μ = 0, we do get the flat space metric, as desired. Let’s look at it the other way. We’ll use the original Einstein equation and plug in our explicit expression for the curvature.



Let’s go one at a time. The time equation is:



Well, this is a Bernoulli equation. So we’ll make the customary substitution to translate it into a linear equation.



We can conclude c = -2μ, if it’s to match up with the weak field limit. So:



So there’s that one. Now let’s get time. I guess this will come from the radial component:



Filling in R, we have:



Integrating,



And eC must be -c2 if the weak-field limit is to be obtained. So we have:



So we recover our results. So actually, doing it this way was pretty easy, despite seemingly being more difficult.

**Interpretation of the time coordinate and radial coordinate**

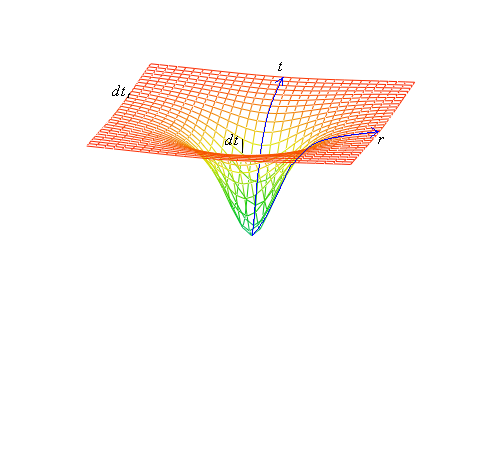
So the time-like coordinate does not represent actual time, and neither does the radial coordinate represent radial distance. Consider a stationary observer out at infinity. We’ll note that as r → ∞, g00 → -1. This tells us that physical time between events A and B for an observer relates to the coordinate time between those events according to [where the implicit xα(s) is the path of our stationary observer]:



and so basically, the physical time for an infinitely far removed person is the same as book-keeping time. We’ll also observe time dilation. Consider an event that begins and ends at book-keeping time tA,B respectively. What is the physical time interval as measured by someone at radial position r? This would be:



So the same event will seem to take less and less time, the further you go into the well. This is just like in the movie Interstellar, but unlike that Star Trek Voyager episode. We can think of this as because the time coordinate is getting stretched out as we go into the well, kind of illustrated below:



We’ll note that as the observer gets close to r = 2μ, events which take no time at all for him, will take close to ∞ time for infinitely far removed observers. This is the event horizon, and the radius is called the Schwarzild radius.



Of course for this formula to apply, this radius must be outside the star. So we could say that if a star’s radius were less than this value, then it would possess a so-called event horizon. The required radius for the Earth, for example, would be about 2mm. Speaking of radii, we should note that this isn’t the actual ‘radius’ as in distance from center of star to event horizon. Radial distances would be calculated via:



Now we don’t actually know the metric inside the star, so we can’t exactly calculate *this*. But outside the star we’d have:



This has some result I won’t bother with. But we’ll note that for rA say somewhat close to the mass and rB far far away, this will just about be rB – rA because the integrand will be approximately 1 for most of the integral. So basically, far from the star, r because the physical radius just as t becomes the physical time.